Approximation of hitting times for stochastic differential equations

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The study and approximation of hitting times of particular boundaries or given thresholds for stochastic processes, are challenging questions and play a central role in numerous applied fields, as in geophysics, neurosciences and finance. For example, in finance they may be used to quantify default risk in path-dependent derivatives.
We will develop new techniques allowing to approximate these hitting times without using time-splitting procedures, like the Euler scheme. We will start by introducing the random Walk on Spheres (WOS) method for the Brownian motion, connected to the heat equation, which exploits the isotropy property. Furthermore, we will construct non-linear boundaries for which we are able to obtain the explicit form of the distribution of the hitting time. Combining this with the connexion between the Bessel process and the Brownian motion will permit to construct a generic algorithm for both the hitting time and the corresponding hitting position of the process. These methods, called Walk on Moving Spheres (WOMS) methods, apply well in some particular cases as they give a good approximation for Bessel processes and some classes of stochastic differential equations.
We will extend these results to multidimensional stochastic differential equations and study the numerical approximation of the first exit time $\tau_{\mathcal{D}}$ of a d-dimensional diffusion process $(X_t)_{t\geq 0}$ from a bounded, regular domain $\mathcal{D}$. The process satisfies

$$\mathrm{d}X_t = \nabla \mathcal{U}(X_t, t)\mathrm{d}t + \mathrm{d}B_t, \qquad X_0 \in \mathcal{D},$$

where (B_t) is a d-dimensional Brownian motion and the drift term may depend on both space and time. The proposed method relies on an acceptance-rejection procedure applied to random walk trajectories and introduces truncated spheroids to account for nonzero drift.
Furthermore, we obtain convergence results and present numerical simulations that permit to illustrate the efficiency and accuracy of these methods.
These results are joint works with Samuel Herrmann (Université Bourgogne Europe, Dijon).

Bibliography

[1] Madalina Deaconu and Samuel Herrmann *Approximation of Diffusion Exit Times from Bounded Domains via a Rejection-Based Random Walk on Truncated spheroids.* Preprint (2026). *https://inria.hal.science/hal-05641595v1/document*
[2] Madalina Deaconu and Samuel Herrmann. *Strong approximation of Bessel processes. Methodology and Computing in Applied Probability*, **25**(2), 1387-5841 (2023)
[3] Madalina Deaconu and Samuel Herrmann. *Strong approximation of some particular one-dimensional diffusions. Discrete and Continuous Dynamical Systems. Series B*, **29**(4), 1990–2017 (2024)
[4] Madalina Deaconu and Samuel Herrmann. *Hitting time for Bessel processes-walk on moving spheres algorithm (WoMS). The Annals of Applied Probability*, **23**(6), 2259–2289 (2013)