Parabolic Hardy-Hénon equations: qualitative theory and large time behavior

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This talk is focused on the mathematical theory of the parabolic Hardy-Hénon equation with degenerate diffusion

$$u_t = \Delta u^m + |x|^\sigma u^p, \quad (x,t) \in \mathbb{R}^N \times (0,\infty),$$

in the range of exponents $m > 1$, $p > 1$ and $\max\{-2,-N\} < \sigma < 0$. The main difficulty in the analysis stems from the presence of the singular coefficient of the source term, and the goal of this talk is to present and discuss its influence. In particular, while the superlinear source induces finite time blow-up when $\sigma = 0$, whatever the value of $p > 1$, at least for sufficiently large initial conditions, a striking effect of the singular potential $|x|^\sigma$ is the prevention of finite time blow-up for suitably small values of p, namely,

$$1 < p \leq p_G := \frac{2 - \sigma(m-1)}{2}.$$

Well-posedness in optimal Lebesgue spaces for whatever value of $p > 1$ and this global existence of any solution if $1 < p \leq p_G$ are obtained by employing the celebrated Caffarelli-Kohn-Nirenberg (CKN) inequalities. Another interesting feature is that uniqueness and comparison principle hold true for generic non-negative initial conditions when $p > p_G$, but are restricted to initial conditions which are positive in a neighborhood of $x = 0$ when $p \in (1, p_G)$, since non-uniqueness holds true in the latter range without this positivity assumption. We also give the large time behavior of solutions as $t \to \infty$ in the range $p \in (1, p_G)$ and discuss conditions for the alternative between finite time blow-up and global existence if $p > p_G$, as well as extensions and open problems related to Hardy-Hénon equations.

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