Existence and boundedness of solutions to singular anisotropic elliptic equations

Florica Cîrstea

School of Mathematics and Statistics, The University of Sydney, NSW 2006, Australia

E-mail address: `florica.cirstea@sydney.edu.au`

Abstract. In this talk, we present new results on the existence and uniform boundedness of solutions for a general class of Dirichlet anisotropic elliptic problems of the form

$$-\Delta_{\overrightarrow{p}} u + \Phi_0(u, \nabla u) = \Psi(u, \nabla u) + f \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where Ω is a bounded domain in $\mathbb{R}^N$ $(N \geq 2)$, $\Delta_{\overrightarrow{p}} u = \sum_{j=1}^N \partial_j(|\partial_j u|^{p_j-2}\partial_j u)$ and $\Phi_0(u, \nabla u) = \left(\mathfrak{a}_0 + \sum_{j=1}^N \mathfrak{a}_j |\partial_j u|^{p_j}\right)|u|^{m-2}u$, with $\mathfrak{a}_0 > 0$, $m, p_j > 1$, $\mathfrak{a}_j \geq 0$ for $1 \leq j \leq N$ and $N/p = \sum_{k=1}^N (1/p_k) > 1$. We assume that $f \in L^r(\Omega)$ with $r > N/p$. The feature of this study is the inclusion of a possibly singular gradient-dependent term $\Psi(u, \nabla u) = \sum_{j=1}^N |u|^{\theta_j-2}u\,|\partial_j u|^{q_j}$, where $\theta_j > 0$ and $0 \leq q_j < p_j$ for $1 \leq j \leq N$.

This is joint work with Barbara Brandolini (Università degli Studi di Palermo).