*A plea for applied mathematics**

Over time, the name of applied mathematics has created confusion even among mathematicians, who have not given a single answer to the question of what applied mathematics is. It is said that ten mathematicians asked this can give fifteen different answers! Analyzing more carefully, we will try to characterize in the sequel this younger direction of mathematics.

We can remember, smiling, the classical definition of Professor Gr. C. Moisil who compared the pure mathematician with the applied one, saying that the former does what he can as it should, and the latter does what it should as he can. A standard definition would be that applied mathematics is a branch of mathematics that deals with the study of mathematical techniques that can be used to apply mathematical knowledge in other scientific and technical fields.

Perhaps these characterizations of applied mathematics, the first, slightly malicious and the second, conventional, could have been correct in the beginning. We will present some arguments to show, however, that applied mathematics is not limited to this, but that it is essentially mathematics, an extensive, deep and creative subdomain of mathematics, which has a dynamic free boundary with it.

In fact, if we look at Europe, we notice an extraordinary breadth of applied mathematics in schools with traditions in pure mathematics, such as France, England and especially Italy. In Italy, important researchers in the field of analysis, former students of the famous school of analysis of Guido Stampacchia in Rome or Enrico Magenes in Pavia, proudly call themselves applied mathematicians. Even professors of analysis at the University of Bologna, recognized for its long tradition in pure mathematics, with extremely abstract theoretical results, complete them with applications to models from other sciences and call this applied analysis.

For the beginning, a list of a few questions might give an idea of what applied mathematics aims to study.

- *Can soil moisture, depending on rainfall intensity, outside temperature and the amount of water taken from vegetation soil, be predicted?*
- *Can the effects of pollutants that are dispersed in air or water be captured and can their trajectory or location of the initial source be determined?*
- *How can knowledge about the interactions between oceans, atmosphere and living ecosystems be combined into models to predict long-term climate change?*

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• How can the spread of a forest fire be mathematically modeled according to the weather, the soil and the type of trees?

• Can math help preventing natural disasters, such as landslides, by optimally determining the location of a natural dam?

• Can the interaction between a predatory population and a population that represents its food be controlled?

• How could an epidemic spread to populated areas and how could it be controlled?

• How can the action of a medicine be improved?

• Could blood pressure be optimized in case of venous insufficiency?

• Can clinical decisions based on a personalized approach to medications be helped?

• How can an investment be allocated between different financial instruments to cope with a risk / return trade-off?

• Can the characteristics of the materials used in construction be determined to optimize the temperature inside a building?

At first glance, these questions do not address mathematics. Nevertheless, applied mathematics can answer and, at least, give useful information for making decisions. It should be noted that some of these questions have been already answered by applied mathematicians. By constructing theoretical objects that are the rigorous mathematical models (as opposed to observation-based empirical models used in science), the characteristics of these processes and phenomena and their interactions can be transposed into the abstract framework of mathematics, and thus studied with its specific tools. Mathematics is seen from the outside as an abstract science, but applied mathematics can provide interpretations of mathematical results for a broad meaning. Practically, any field of research, physics, chemistry, engineering, economics, sociology, biology, medicine can benefit from the involvement of mathematics.

In applied mathematics, researchers seek to understand concepts in other fields and choose appropriate mathematical techniques to analyze them, while examining the limitations of models and solving techniques and developing them to allow an effective approach. Applied mathematics must be seen as mathematics adapted to real-world problems with the dual purpose of explaining observed phenomena and predicting new, possibly yet unobserved behaviors.
Therefore, the emphasis is on developing improved methods or creating new ones to meet the challenges of new problems and the real world.

But not every intervention of mathematics in other fields can be called applied mathematics. A clearer definition of it includes some conditions that can be summarized as follows:

(i) creating the mathematical model that describes the phenomenon or process by establishing a mathematical formulation based on equations or probabilistic methods;

(ii) the mathematical analysis of the respective model, with adequate mathematical methods, for the rigorous proof of the existence, uniqueness, regularity and other properties of the solution;

(iii) numerical analysis, creation of algorithms and codes and numerical simulation;

(iv) comparison between theoretical and experimental results and interpretations in order to improve models or their calibration.

The mathematics used to make them combines knowledge from the theory of differential equations, partial differential equations, equations of evolution in infinitely dimensional spaces, functional analysis, operator theory, stochastic processes, probabilities, real analysis, complex analysis, measure theory, convex analysis, numerical methods, thus involving many fields of mathematics.

It is true that certain researchers give a greater weight only to some of the aspects (i) - (iv). Some of them introduce only the model, make numerical simulations based on existing software and possibly present comparisons with observed data. Others insist on the development of a specific numerical technique and on the creation of their own algorithm for solving the problem, obtaining a good and necessary result. But research that does not take into account model analysis and a rigorous proof that it is well-posed is not enough for a mathematician.

We will refer to the researches that, in the point of view (ii), focus on the theoretical mathematical part itself, by investigating the models (proposed or taken from the literature) from a theoretical point of view. Essential for the mathematician is this conceptual aspect to prove first of all that the model makes sense, i.e., that it has at least one solution, possibly unique and with properties in accordance with the behavior of the studied process. An extremely important action here consists in defining new concepts and creating new theoretical mathematical results to face the difficulty of new mathematical models which, trying to capture as much as possible of the complexity of physical phenomena, turn out in being practically unsolvable from the point of view of their mathematical consistency (well-posedness). Their simplification, which would allow to prove a possible solution, would lead to less correct characterizations of the studied processes. The ability and knowledge of the applied mathematician consists precisely in extracting the most appropriate knowledge from theory and in creating new mathematical tools to overcome the difficulties of a more precise model but for which there are no precedents in solving.

Regarding the first, extremely difficult part, specifically the modeling, which provides the applied mathematician a clearer understanding of the process to be investigated, this is absent in many papers. It can be correctly done via an interdisciplinary research. Thus, other sciences offer new challenges that lead to the creation of new theoretical results in mathematics, in addition to the results it can provide to partner science. Therefore, the relationship between (applied) mathematics and other sciences is not of subordination but of collaboration for the benefit of each of them.

We agree that, in the context of the analysis and control of mathematical models in the real world, we call here applied mathematics a complete research defined as above, but which is necessarily able to produce new theoretical results in the basic mathematical fields: theory of partial differential equations, theory of infinitely dimensional systems, theory of semigroups, theory of control, theory of probabilities, stochastic equations, functional analysis. We conclude that applied mathematics is mathematics in itself, it does not discover the laws of nature, but through its own analysis of models it can lead to new scientific discoveries.

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